Code No: 09A1BS01 **R09** **Set No. 2**

I B.Tech Examinations,June 2011
MATHEMATICS-1
Common to CE, ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, AIE, ICE, E.COMP.E, MMT, ETM, EIE, CSE, ECE, EEE

Time: 3 hours Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Find the radius of curvature for the curve $\sqrt{a} = \sqrt{r}\cos\theta/2$ at $\theta = \Pi/2$
   (b) Find the envelop of the straight line $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are connected by the relation $a + b = c$ where c is a constant. [8+7]
2. (a) Find the directional derivative of $f(x,y,z)=zx^2-xyz$ at the point (1,3, 1) in the direction of the vector $3i - 2j + k$.
   (b) Evaluate the line integral $\int_c (x^2 + xy)\,dx + (x^2 + y^2)\,dy$ where c is the square formed by the lines $y = \pm 1$ and $x = \pm 1$. [8+7]
3. (a) Test the convergence of the series $\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \ldots\infty$
   (b) Test the convergence of the series $\frac{3x}{4} + \left(\frac{5}{6}\right)^2 x^2 + \left(\frac{7}{8}\right)^3 x^3 + \ldots..$ [7+8]
4. (a) Solve the differential equation $(D^2 + 4)y = x \sin x$
   (b) Solve by method of variation of parameters $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ [7+8]
5. (a) If $x + y + z = u$, $y + z = uv$, $z = uvw$ show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$
   (b) Divide 24 into three points such that the continued product of the first, square of the second and cube of the third is maximum. [8+7]
6. (a) The arc of the cardioid $r = a\,(1+\cos\theta)$ included between $\theta = {-\pi}/{2}$ and ${\pi}/{2}$ is rotated about the line $\theta = {\pi}/{2}$. Find the surface area of the solid generated.
   (b) Evaluate by changing the order of integration $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$ [8+7]
7. (a) Find $L[t \sin 3t \cos 2t]$
   (b) Solve the following differential equation using the Laplace transforms $\frac{d^2y}{dt^2} + \frac{2dy}{dt} + 2y = 5\sin t \quad y(0) = y^1(0) = 0$ [8+7]
8. (a) Form the differential equation by eliminating arbitrary constants $y = Ae^{-3x}+Be^{2x}$
   (b) Solve the differential equation $(y - x^2)dx + (x^2 \cot y - x)dy = 0$
   (c) Find the equation of the curve, in which the length of the subnormal is proportional to the square of the abscissa. [4+6+5]

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Time: 3 hours Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Find the volume of the solid generated by the revolution of the curve (a-x) $y^2 = a^2 x$ about its asymptote.
   (b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dxdydz$ [8+7]

2. (a) Test the convergence of the series $\sum \frac{1}{n} \log \frac{n+1}{n}$
   (b) Test the convergence of the series $\sum \frac{(n+1)^n x^n}{n^{n+1}}$ [7+8]

3. (a) Find L $\frac{\cos 4t \sin 2t}{t}$
   (b) Find the Laplace inverse transform of log $\frac{s^2+4}{s^2+9}$
   [7+8]

4. (a) Form the differential equation by eliminating arbitrary constants $xy = Ae^x + Be^{-x}$
   (b) Solve the differential equation $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy$
   (c) If the air is maintained at $25^0$ and the temperature of the body cools from 140 $^0$C to $80^0$C in 20 minutes, find when the temperature will be $35^0$ [4+6+5]

5. (a) In what direction from the point (-1, 1, 2) is the directional derivative of $\varphi = xy^2z^3$ a maximum what is the magnitude of this maximum.
   (b) Fnd the circulation of $\overline{F}$ round the curve c where $\overline{F} = (e^x \sin y)\, i + (e^x \cos y)\, \mathbf{j}$ and c is the rectangle whose vertices are $(0, 0)\,(1, 0)\,(1, \pi/2)\,, (0, \pi/2)$. [8+7]

6. (a) Expand $e^{x \sin x}$ in powers of x.
   (b) Find the volume of the greatest rectangular parallelopiped that can be 'inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. [8+7]

7. (a) Solve the differential equation $(D^2 - 4)y = 2\cos^2 x$
   (b) A particle is executing S.H.M, with amplitude 5 meters and time 4 seconds. Find the time required by the particle in passing between points which are at distances 4 and 2 meters from the centre of force and are on the same side of it. [8+7]

8. (a) The radius of curvature at any point P on the parabola $y^2 = 4ax$ and S is the focus, then prove that $\rho^2 \alpha$ (SP $)^3$
   (b) Find the equation of the circle of curvature of the curve $x = a(\cos\theta + \theta \sin\theta)$, $y = a(\sin\theta + \theta\cos\theta)$ [7+8]

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Code No: 09A1BS01 **R09** **Set No. 1**

I B.Tech Examinations,June 2011

MATHEMATICS-1

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Time: 3 hours Max Marks: 75

Answer any FIVE Questions

All Questions carry equal marks

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1. (a) Prove that if $\varphi$ and $\psi$ are scalar functions. Then prove that $\nabla\varphi \times \nabla\psi$ is solenoidal.
   (b) Find whether the function $\bar{F} = (x^2 - y^3)\,i + (y^2 - 3x)\,j + (z^2 - xy)\,k$ is irrotational and hence find scalar potential function corresponding to it. [8+7]

2. (a) If $x = u(1 - v), y = uv$ prove that $JJ^1 = 1$
   (b) Find the rectangular parallelepiped of maximum volume that can be inscribed in a sphere. [7+8]

3. (a) Find the envelope of $x \operatorname{cosec}\theta - y \cot\theta = p$ where $\theta$ is a parameter.
   (b) Trace the curve $r = a\,(1 - \cos\theta)$ [7+8]

4. (a) Form the differential equation by eliminating arbitrary constants $y = a\,x^3 + bx^2$
   (b) Solve the differential equation $x^3 \frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2}$
   (c) Find the orthogonal Trajectories of the family of curves $x^2+y^2 = a^2$ [4+6+5]

5. (a) Find $L\,[e^{-3t} \sinh 3t]$ using change of scale property
   (b) Find the Laplace inverse transform of $\frac{s+3}{(s^2+6s+13)^2}$ [8+7]

6. (a) Test the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots..\infty$
   (b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1.3.5.\cdots(2n+1)}{2.5.8\cdots(3n+2)}$ [7+8]

7. (a) Find the perimeter of the loop of the curve $3ay^2 = x^2(a - x)$
   (b) Evaluate $\int_0^{\pi/2}\int_0^{\infty} \frac{r\,dr\,d\theta}{(r^2+a^2)^2}$ [8+7]

8. (a) Solve the differential equation $(D^3 + 2D^2 + D)y = e^{2x}$
   (b) A body weighing 10kgs is hung from a spring. A pull of 20 kgs will stretch the spring to 10 cms. The body is pulled down to 20 cms below the static equilibrium position and then released.Find the displacement of the body from its equilibrium position at time t seconds, the maximum velocity and the period of oscillation. [8+7]

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Code No: 09A1BS01 **R09** **Set No. 3**

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Time: 3 hours Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$
   (b) Test the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots . \infty$ [7+8]
2. (a) Solve the differential equation $(D^3 - 1)y = (e^x + 1)^2$
   (b) Solve the differential equation $(D^4 + 2D^2 + 1)y = x^2 \cos^2 x$ [8+7]
3. (a) Find a unit normal vector to the surface $x^3+y^3+3xyz=3$ at the point (1, -2, -1).
   (b) Evaluate by stokes theorem $\int_c (e^x dx + 2ydy - dz)$ where c is the curve $x^2+y^2 = 9$ and $z = 2$ [8+7]
4. (a) Find the differential equation of all circles whose radius is r
   (b) Solve the differential equation $(x + 1)\frac{dy}{dx} - y = e^{3x}(x + 1)^2$
   (c) Find the equation of the curve, in which the length of the subnormal is proportional to the square of the ordinate. [4+6+5]
5. (a) If $L[f(t)] = \bar{f}(s)$, then prove that $L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} \bar{f}(s)\, ds$ provided $\lim_{t \to 0} \frac{f(t)}{t}$ exists.
   (b) $\frac{s+3}{s^2-10s+29}$ [8+7]
6. (a) Find the whole area of the lemniscates $r^2 = a^2 \cos 2\theta$
   (b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{\frac{a^2-r^2}{2}} rdzdrd\theta$. [8+7]
7. (a) Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using lagranges mean value theorem.
   (b) Expand $e^y \log(1 + x)$ in powers of x,y. [8+7]
8. (a) Show that the evolute of the ellipse $x = a\cos\theta, y = b\sin\theta$ is $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$
   (b) Show that the envelope of the lines whose equations are $x\sec^2\theta + y\,\mathrm{cosec}^2\theta = c$ is a parabola which touches the axes of coordinates. [8+7]

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