Code No: 09A1BS01 **R09** **Set No. 2**

I B.Tech Regular Examinations,June 2010
MATHEMATICS-1
Common to ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE, E.COMP.E, MMT, ETM, EIE, CSE, ECE, EEE,CE

Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\sqrt{n+1}-\sqrt{n-1}$
   (b) Test the convergence of the series $u_n = \frac{(n+1)^n . x^n}{n^{n+1}}$
   (c) For what values of the following series is convergent $-x+\frac{x^2}{2^2}-\frac{x^2}{2^2}+\frac{x^3}{3^2}-\frac{x^4}{4^2}+\ldots$ [5+5+5]

2. (a) Find the radius of curvature at the origin for the curve $x^4-y^4+x^3-y^3+x^2-y^2+y=0$
   (b) Find the centre of curvature on $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ [8+7]

3. (a) Find a unit normal vector to the surface $x^3+y^3+z^3=3$ at the point $(1,-2,1)$
   (b) Applying, Green's theorem evaluate $\int (y-\sin x)\,dx+\cos x dy$, where C is the plane triangle enclosed by the lines $y=0, x=\frac{\pi}{2}$ *and* $y=\frac{2x}{\pi}$ [8+7]

4. (a) Using mean value theorem, For $0<a<b$ Prove that $1-\frac{a}{b}<\log\frac{b}{a}<\frac{b}{a}-1$ and hence show that $\frac{1}{6}<\log\frac{6}{5}<\frac{1}{5}$
   (b) Expand $e^x\sin y$ in powers of x and y. [8+7]

5. (a) Find the length of the cycloid x = a (θ + sinθ) y = a (1-cosθ) between two consecutive cusps. Show that the length of the arc of the cycloid between the points $\theta=0$ and $\theta=2\Psi$ is given by s = 4asin ψ. Show further that for this curve $s=\sqrt{8ay}$.
   (b) Evaluate the double integral $\int_0^a\int_0^{\sqrt{a^2-y^2}}(x^2+y^2)\,dy\,dx$ [7+8]

6. (a) Find $L\left[g(t)\right]$ where $\begin{aligned} g(t) &= \cos\left(t-\tfrac{2}{3}\pi\right) t > {}^{2\pi}/_{3} \\ &= 0,\ t < {}^{2\pi}/_{3} \end{aligned}$
   (b) Find $L^{-1}\left[\frac{s}{(s^2+1)(s^2+9)(s^2+25)}\right]$ [8+7]

7. (a) Form the differential equation by eliminating arbitrary constants $y = C\,e^{\sin^{-1}x}$
   (b) Solve the differential equation $(x^2+2y^2)dx-xydy=0$
   (c) If the air is maintained at 25^0C and the temperature of the body cools from 100^0 to 80^0C in 10 minutes, find the temperature of the body after 20 minutes and when the temperature will be 40^0C. [3+6+6]

8. (a) Solve the differential equation $(D^2+9)y=\cos 3x+\sin 2x$

(b) A mass of 4 lbs suspended from a light elastic string of natural length 3 feet extends it to a distance 2 ft. One end of the string is fixed and a mass of 2 lbs is attached to other. The mass is held so that the string is just un stretched and is then let go. Find the amplitude, period and the maximum velocity of the S.H.M. [8+7]

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Time: 3 hours **Max Marks: 75**

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1. (a) Prove that div $(r^n \bar{r}) = (n+3)\, r^n$. Hence show that $\frac{\bar{r}}{r^3}$ is solenoidal
   (b) If $\bar{F} = (x+y+1)\, i + j - (x+y)\, k$, then show that $\bar{F}$ curl $\bar{F}$=0 [7 +8]
2. (a) Form the differential equation by eliminating arbitrary constants $\log \frac{y}{x} = C$
   (b) Solve the differential equation $\frac{dy}{dx} = e^{2x-3y} + x^2 e^{-3y}$
   (c) If the air is maintained at 30 ^{0}C and the temperature of the body cools from 80 ^{0}C to 60 ^{0}C in 12 minutes, find the temperature of the body after 24 minutes. [4+5+6]
3. (a) Find $L\left[e^{-3t} \sinh 3t\right]$ using change of scale property.
   (b) Solve the following differential equation using the Laplace transforms $y^{11} -$ y ^{11}_ 3y^{1} + 2y = 4t + e^{3t}, y(0) = y^{1}(0) = 1. [8+7]
4. (a) Test the convergence of the series $\frac{3^2}{6^2} + \frac{3^2.5^2}{6^2.8^2} + \frac{3^2.5^2.7^2}{6^2.8^2.10^2} + ...$
   (b) Find the interval of convergence for the following series $\frac{\sum (-1)^n (x+2)}{(2^n+5)}$ [7+8]
5. (a) If ρ_1 and ρ_2 are radii of curvature at the extremities of any chord of the cardioids $r = a\,(1+\cos\theta)$, which passes through the pole, then show that $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$
   (b) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$. Where $a^2 + b^2 = c^2$ [8+7]
6. (a) Expand $e^{x \sin x}$ in powers of x.
   (b) Find the volume of the greatest rectangular parallelopiped that can be 'inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. [8+7]
7. (a) Find the Volume of the solid obtained by revolving one loop of the curve $r^2 = a^2 \cos^2\theta$ about the line $\theta = {}^{\pi}/_{2}$.
   (b) Evaluate by changing the order of integration $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dy dx}{\sqrt{y^4 - a^2x^2}}$ [7+8]
8. (a) Solve the differential equation $(D^2 - 4)y = 2\cos^2 x$
   (b) A particle is executing S.H.M, with amplitude 5 meters and time 4 seconds. Find the time required by the particle in passing between points which are at distances 4 and 2 meters from the centre of force and are on the same side of it. [8+7]

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1. (a) Find the radius of curvature at any point on $y^2 = 4ax$ and hence show that the radius of curvature at the vertex is equal to the semi latus rectum.
   (b) Trace the curve $r = a\,(1 + \cos\theta)$ [7+8]

2. (a) Find the volume of Spherical cap of height h cut off from a sphere of radius a.
   (b) Evaluate $\int\limits_0^{\pi} \int\limits_0^{a(1+\cos\theta)} r^2 Cos\theta dr d\theta$ [8+7]

3. (a) Solve the differential equation $(D^2 + D + 1)y = x^3$
   (b) Solve the differential equation $(D^2 + 1)y = \sin x \sin 2x$ [8+7]

4. (a) Form the differential equation by eliminating arbitrary constants y = a x^3+bx^2
   (b) Solve the differential equation $x^3 \frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2}$
   (c) Find the orthogonal Trajectories of the family of curves $x^2+y^2 = a^2$ [4+6+5]

5. (a) If $u = x^2 - 2y, v = x + y + z,\ w = x - 2y + 3z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$
   (b) Find the maximum and minimum values of $f(x) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ [8+7]

6. (a) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)\,x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (-1,1,2).
   (b) Evaluate $\oint\limits_e (yzdx + xzdy + xydz)$ over arc of a helix $x = a\cos t, y = a\sin t, z = kt\ as\ t$ varies from 0 to 2π [8+7]

7. (a) Find $L\left[\frac{e^{-t}\sin t}{t}\right]$
   (b) Solve the following differential equation using the Laplace transforms $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^{2t}$ with $x(0) = 2,\ \frac{dx}{dt} = -1\ at\ t = 0$ [8+7]

8. (a) Test the convergence of the series $\frac{(n!)^2x^{2n}}{(2n)!}$
   (b) Test the convergence of the series $\sum \frac{\left(\sqrt{5}-1\right)^n}{n^2+1}$ [7+8]

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Time: 3 hours **Max Marks: 75**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Solve the differential equation $(D^4 - 2D^3 + 2D^2 - 2D + 1)y = \cos x$
   (b) Solve the differential equation $(D^3 - 3D - 2)y = x^2$ [7+8]

2. (a) Form the differential equation by eliminating arbitrary constants y = e^x (ACos x + B Sin x)
   (b) Solve the differential equation $e^{x-y}dx + e^{y-x}dy = 0$
   (c) If the air is maintained at 15^0C and the temperature of the body drops from 70^0 C to 40^0 in 10 minutes. What will be its temperature after 30 minutes. [4+5+6]

3. (a) If $u^3 + xv^2 - uy = 0$, $u^2 + xyv + v^2 = 0$ find $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$
   (b) Find the shortest distance from the point (1,0) to the parabola $y^2 = 4x$ [8+7]

4. (a) Find the directional derivative of f(x,y,z)=xy^2+yz^3 at the point (2,-1,1) in the direction of the vector i+2j+2k.
   (b) Evaluate by stoke's theorem $\int\limits_C (e^x dx + 2ydy - dz)$ where c is the curve $x^2 + y^2 = 9 \; and \; z = 2$ [8+7]

5. (a) Find the volume of the solid generated by cycloid x = a (θ + sinθ), y = a (1+cosθ), when it is revolved about its base.
   (b) Evaluate $\int_0^{\log z} \int_0^x \int_0^{x+\log y} e^{x+y+z} dzdydx$ [8+7]

6. (a) Find the Laplace transform of periodic function f(t) with period T, where $f(t) = \frac{4Et}{t} - E, \; 0 \leq t \; \leq {}^{T}\!/_{2} = 3E - \frac{4E}{T}t, \; \frac{T}{2} \leq t \leq T$
   (b) Find the inverse Laplace transform of $\frac{(2s^2-6s+5)}{(s^3-6s^2+11s-6)}$ [8+7]

7. (a) Test the convergence of the series $\frac{1}{3} + \frac{1.4}{3.6} + \frac{1.4.7}{3.6.9} + \frac{1.4.7.10}{3.6.9.12} + ...$
   (b) Prove that the series $\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) .....\infty$ is conditionally convergent. [7+8]

8. (a) The radius of curvature at any point P on the parabola $y^2 = 4ax$ and S is the focus, then prove that $\rho^2 \alpha \; (SP)^3$
   (b) Find the equation of the circle of curvature of the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta + \theta\cos\theta)$ [7+8]

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