

I B.Tech Examinations, December-January, 2011-2012

MATHEMATICAL METHODS

Common to BME, IT, ICE, E.COMP.E, ETM, EIE, CSE, ECE, EEE

Time: 3 hours

Max Marks: 75

Answer any FIVE Questions

All Questions carry equal marks

1. Find the eigen values and the corresponding eigen vectors of $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. [15]
2. (a) Find the value of k such that the rank of $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & k & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is 2.
- (b) Find whether the following system of equations are consistent. If so solve them. $2x - y + z = 5$, $3x + y - 2z = -2$, $x - 3y - z = 2$. [7+8]
3. (a) Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$.
- (b) Find the integral surface of $x(y^2 + z)p - y(x^2 + z)q = (x^2 + y^2)z$. [7+8]
4. Given $y' = x + \sin y$ and $y(0) = 1$ compute $y(0.2)$ and $y(0.4)$ with $h = 0.2$ using Euler's modified method. [15]
5. Reduce the quadratic form to the canonical form $3x^2 - 3y^2 - 5z^2 - 2xy - 6zx - 6yz$. [15]
6. Obtain the half-range sine and cosine series for the function $f(x) = \frac{\Pi x}{8} (\Pi - x)$ in the range $0 \leq x \leq \Pi$. [15]
7. (a) Derive the formula to evaluate $\int_a^b y dx$ using Simpson's $\frac{1}{3}$ rule.
- (b) Evaluate $\int_{0.6}^2 y dx$ using Trapezoidal rule. [8+7]
8. (a) Find a real root of the equation $3x = \cos x + 1$ by bisection method.
- (b) Given that $y(3) = 6$, $y(5) = 24$, $y(7) = 58$, $y(9) = 108$, $y(11) = 174$ find x when $y = 100$, Using Lagranges formula. [8+7]

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1. (a) Solve $y' = x^2 + y^2$, $y(0)=1$ using picard's method.
 (b) Solve $y' + y = e^x$, $y(0)=0$ using picard's method. [7+8]

2. (a) Fit a straight line $y=a+bx$ from the following data:

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

- (b) Fit a straight line to the form $y=a+bx$ for the following data:

x	0	5	10	15	20	25
y	12	15	17	22	24	30

[8+7]

3. (a) Find a real root of the equation $2x - \log x = 7$ using iteration method
 (b) Find $y(54)$ given that $y(50)=205$, $y(60)=225$, $y(70)=248$ and $y(80)=274$ Using Newton's forward difference formula. [8+7]

4. (a) Find the rank of the Matrix by reducing it to the normal form.

$$\begin{bmatrix} 4 & 3 & 2 & 1 \\ 5 & 1 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 1 & -1 & 3 & -2 \end{bmatrix}$$

- (b) Solve the following tridiagonal system $3x - y = 4$, $2x - y + z = 6$, $2y + 3z + 2w = 11$, $z - 2w = -1$. [8+7]

5. Verify Cayley Hamilton theorem and find the inverse of $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. [15]

6. Reduce the quadratic form to the canonical form $2x^2 + 2y^2 + 2z^2 - 2xy + 2zx - 2yz$. [15]

7. (a) Solve $(p^2 - q^2)z = x - y$.
 (b) Solve $px - qy = y^2 - x^2$. [8+7]

8. If $f(x) = \begin{cases} \frac{\Pi x}{4} & \text{for } 0 < x < \frac{\Pi}{2} \\ \frac{\Pi}{4}(\Pi - x) & \text{for } \frac{\Pi}{2} < x < \Pi \end{cases}$. Show that

(a) $f(x) = \sin x - \frac{1}{3^2} \sin 3x + \frac{1}{5^2} \sin 5x - \frac{1}{7^2} \sin 7x + \dots$

(b) $f(x) = \frac{\Pi^2}{16} - \frac{1}{2} \left(\frac{1}{1^2} \cos x + \frac{1}{3^2} \cos 3x + \dots \right)$. [15]

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1. (a) Form the partial differential equation by eliminating f from $xyz=f(x^2+y^2+z^2)$.
 (b) Form the partial differential equation by eliminating f from $z=(x+y)f(x^2-y^2)$. [7+8]
2. (a) Prove that the eigen vectors corresponding to two different eigen values are linearly independent.
 (b) Find the eigen values and the corresponding eigen vectors of $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 2 & 2 \\ -1 & -1 & 0 \end{bmatrix}$. [6+9]
3. Reduce the quadratic form to the canonical form $3x^2+2y^2+3z^2-2xy-2yz$. [15]
4. (a) Find a real root of the equation $\cos x - x^2 - x = 0$ using Newton Raphson method.
 (b) Find the second difference of the polynomial $x^4-12x^3+42x^2-30x+9$, given that the interval of difference is 2. [7+8]
5. Given $\frac{dy}{dx}=-xy^2$ and $y(0)=2$. Compute $y(0.2)$ in steps of 0.1 using modified Euler's method. [15]
6. (a) Find the Rank of the Matrix, by reducing it to the normal form. $\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$
 (b) Solve the following equations by expressing the coefficient matrix as a product of a lower triangular and upper triangular matrices. $x + y - z = 5$, $2x + y + 2z = 5$, $3x + 2y - 4z = 7$. [7+8]
7. (a) Find the half-range sine series of $f(x)=1$ in $[0, l]$.
 (b) Find the half-range cosine and sine series for $f(x)=x$ in $(0, l)$. [7+8]
8. (a) Derive the formula to evaluate $\int_a^b y dx$ using trapezoidal rule.
 (b) The table below shows the temperature $f(t)$ as a function of time
- | | | | | | | | |
|------|----|----|----|----|----|----|----|
| t | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| f(t) | 81 | 75 | 80 | 83 | 78 | 70 | 60 |
- Use Simpson's $\frac{1}{3}$ method to estimate $\int_1^7 f(t) dt$. [8+7]

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1. (a) Find the Fourier Series to represent the function
- $f(x)$
- given

$$f(x) = \begin{cases} 0 & \text{for } -\Pi \leq x \leq 0 \\ x^2 & \text{for } 0 \leq x \leq \Pi \end{cases}$$

- (b) Find the Fourier Series in
- $[-\Pi, \Pi]$
- for the function

$$f(x) = \begin{cases} \frac{-1}{2}(\Pi + x) & \text{for } -\Pi \leq x \leq 0 \\ \frac{1}{2}(\Pi - x) & \text{for } 0 \leq x \leq \Pi \end{cases} \quad [8+7]$$

2. (a) Find the nature of the quadratic form
- $x^2 + 2y^2 + 2z^2 - 2xy + 2yz$
- .

- (b) Find the nature of the quadratic form
- $2x^2 + 2y^2 + 2z^2 + 2yz$
- . [8+7]

3. Evaluate
- $y(0.2)$
- and
- $y(0.4)$
- correct to four decimal places by Taylor's series method if
- $y(x)$
- satisfies
- $y' = 1 - 2xy$
- and
- $y(0) = 0$
- . [15]

4. (a) Solve
- $p^2 + q^2 = x^2 + y^2$

- (b) Solve
- $pq + qx = y$
- . [7+8]

5. Verify Cayley Hamilton theorem and find the inverse of
- $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$
- . [15]

6. Evaluate
- $\int_0^1 \frac{1}{1+x} dx$

- (a) By Trapezoidal rule and Simpson's
- $\frac{1}{3}$
- rule.

- (b) Using Simpson's
- $\frac{3}{8}$
- rule. [8+7]

7. (a) Find a real root of the equation
- $x^3 - 2x^2 - 4 = 0$
- using iteration method.

- (b) Find the polynomial which fits the data in the following table using Gauss forward formula.

x	3	5	7	9	11
y	6	24	58	108	174

[8+7]

8. (a) Find the Rank of the Matrix, by reducing it to the normal form.
- $\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$

- (b) Solve the following equations by expressing the coefficient matrix as a product of a lower triangular and upper triangular matrices.
- $2x + y - z = 3$
- ,
- $x - 2y - 2z = 1$
- ,
- $-x + 2y - 3z = 9$
- . [7+8]
