Code No: 09A1BS01 **R09**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD

B. Tech I Year Examinations, December-2012

MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, CHEM, EIE, BME, IT, MCT, ETM, MMT, ECC, ECC, AE, ICE, BT, AME, MIE, MIM, PTME)

Time: 3 hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1.a) i) Test for absolute convergence/conditional convergence of the series $\sum(-1)^{n-1}\left(\frac{1}{4n-3}\right)$.

ii) Find the interval of convergence of the series $\sum \log(1+x)$.

b) Test for convergence of the series:

i) $\sum_{n=1}^{\infty} \frac{e^n}{e^{2n}+1}$.

ii) $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{n^{3/2}}$. [8+7]

2.a) Show that the volume of the largest rectangular parallelepiped that can be inscribed in a ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{8abc}{\sqrt{27}}$.

b) If a < b prove that $\frac{b-a}{1+b^2} < Tan^{-1}b - Tan^{-1}a < \frac{b-a}{1+a^2}$ using Lagrange's Mean Value Theorem. Deduce the following:

i) $\frac{\pi}{4}+\frac{3}{25} < Tan^{-1}\frac{4}{3} < \frac{\pi}{4}+\frac{1}{6}$.

ii) $\frac{5\pi+4}{20} < Tan^{-1}2 < \frac{\pi+2}{4}$. [8+7]

3.a) Find the coordinates of the center of curvature at any point of the parabola $y^2=4ax$. Hence show that it's evolute is $27ay^2=4(x-2a)^3$.

b) Trace the curve $x^3+y^3=3axy$. [8+7]

4.a) Change the order of integration $\int_0^1\int_{x^2}^{2-x} xydxdy$ and find the double integral.

b) Find the volume of revolution of $r^2=a^2\cos 2\theta$ about the initial line.

5.a) Solve the differential equation $x^3 \frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2}$.

b) Show that the system of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ where λ is a parameter, is self orthogonal.

c) Form the differential equation by eliminating arbitrary constants, $y = ax + b$.

[6+

6. a) Solve $\left(x^2D^2 - 3xD + 1\right)y = \frac{\log x \sin(\log x) + 1}{x}$.

b) A mass of 4 1bs suspended from a light elastic string of natural length 3 feet extends it to a distance 2 ft. One end of the string is fixed and a mass of 2 lbs is attached to other. The mass is held so that the string is just un-stretched and is then let go. Find the amplitude period and the maximum velocity of the S.H.M.

[8+7]

7. a) Find Inverse Laplace Transform of $\log\left|\frac{s^2+4}{s^2+9}\right|$.

b) Solve the differential equation using Laplace transform $\frac{d^2x}{dt^2} + 9x = \sin t$, $x(0) = 1$, $x^1(0) = 0$.

[8+7]

8. a) Prove that $grad\left(\bar{F}.\bar{G}\right) = \bar{F}\times\left(\nabla\times\bar{G}\right) + \bar{G}\times\left(\nabla\times\bar{F}\right) + \left(\bar{F}.\nabla\right)\bar{G} + \left(\bar{G}.\nabla\right)\bar{F}$.

b) Find the work done by a force $\bar{F} = \left(x^2 - y^2 + x\right)\bar{i} - \left(2xy + y\right)\bar{j}$, which moves a particle in xy-plane from (0, 0) to (1, 1) along the parabola $y^2 = x$.

[8+7]

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