

I B.Tech Examinations, December 2010

MATHEMATICS-I

Common to CE, ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE,
E.COMPE, MMT, ETM, EIE, CSE, ECE, EEE

Time: 3 hours

Max Marks: 75

Answer any FIVE Questions

All Questions carry equal marks

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1. (a) Find $L \frac{\cos 4t \sin 2t}{t}$
- (b) Find the Laplace inverse transform of $\log \frac{s^2+4}{s^2+9}$ [7+8]
2. (a) Test the convergence of the series $1 + \frac{3x}{7} + \frac{3.6}{7.10} x^2 + \frac{3.6.9}{7.10.13} x^3 + \frac{3.6.9.12}{7.10.13.16} x^4 + \dots$
- (b) Find the interval of convergence for the series $\sum_{n=1}^{\infty} (-1)^n \frac{n(x+1)^n}{2^n}$ [7+8]
3. (a) Find the length of an arc of the curve $x = e \sin \theta$, $y = e \cos \theta$ from $\theta = 0$ to $\frac{\pi}{2}$
- (b) Evaluate $\iint_R y^2 dx dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ [8+7]
4. (a) Find the differential equation of all circles whose radius is r
- (b) Solve the differential equation $(x+1) \frac{dy}{dx} - y = e^{3x} (x+1)^2$
- (c) Find the equation of the curve, in which the length of the subnormal is proportional to the square of the ordinate. [4+6+5]
5. (a) Solve the differential equation $(D^4 + 2D^2 + 1)y = x^2 \cos^2 x$
- (b) Solve the differential equation $(D^2 + 2D + 1)y = e^{-x}$ [7+8]
6. (a) Expand $e^{x \sin x}$ in powers of x.
- (b) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. [8+7]
7. (a) Show that the evolute of the ellipse $x = a \cos \theta$, $y = b \sin \theta$ is $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$
- (b) Show that the envelope of the lines whose equations are $x \sec^2 \theta + y \cos^2 \theta = c$ is a parabola which touches the axes of coordinates. [8+7]
8. (a) Find the work done by the force $\vec{F} = (2y+3)\mathbf{i} + xz\mathbf{j} + (yz-x)\mathbf{k}$ when it moves a particle from the point (0,0,0) to (2,1,1) along the curve $x = 2t^2$, $y = t$ and $z = t^3$
- (b) Use divergence theorem to Evaluate $\iiint_S (y^2 z^2 \mathbf{i} + z^2 x^2 \mathbf{j} + z^2 y^2 \mathbf{k}) \cdot \vec{n} ds$ where S is the part of the unit sphere above the x-y plane. [8+7]

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1. (a) Find $L [(t^2 + 1)^2]$
 (b) Find Inverse Laplace transform of $\frac{3s+7}{(s^2-2s-3)}$ [7+8]
2. (a) Find the radius of curvature at any point on $y^2 = 4ax$ and hence show that the radius of curvature at the vertex is equal to the semi latus rectum.
 (b) Trace the curve $r = a(1 + \cos \theta)$ [7+8]
3. (a) Test the convergence of the series $\frac{3^2}{6^2} + \frac{3^2 \cdot 5^2}{6^2 \cdot 8^2} + \frac{3^2 \cdot 5^2 \cdot 7^2}{6^2 \cdot 8^2 \cdot 10^2} + \dots$
 (b) Test whether the following series is absolutely convergent or conditionally convergent $\frac{1}{5 \cdot 2} - \frac{1}{5 \cdot 3} + \frac{1}{5 \cdot 4} \dots (-1)^n \frac{1}{5 \cdot n}$ [7+8]
4. (a) Solve the differential equation $(D^2 + 2)y = e^x \cos x$
 (b) Solve the differential equation $(D^3 + 2D^2 + D)y = x^3$ [7+8]
5. (a) If $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + 3z$ find $\frac{(u,v,w)}{(x,y,z)}$
 (b) Find the maximum and minimum values of $f(x) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ [8+7]
6. (a) The curve $y^2 (a + x) = x^2 (3a - x)$ revolved about the x-axis. Find the volume of the solid generated.
 (b) Evaluate $\int_0^R \int_0^{\sqrt{R^2-y^2}} (x^2 + y^2) dx dy$ by changing into polar coordinates [8+7]
7. (a) Form the differential equation by eliminating arbitrary constants $y = Ae^x + Be^{-x}$
 (b) Solve the differential equation $(e^y + 1) \cos x dx + e^y \sin x dy = 0$
 (c) Find the curve in which the perpendicular upon the tangent from the foot of the ordinate of the point of contact is constant and equal to a. [4+6+5]
8. (a) If \vec{F} and \vec{G} are two vectors, then prove that $\text{div } (\vec{F} \times \vec{G}) = \vec{F} \cdot \text{curl } \vec{G} - \vec{G} \cdot \text{curl } \vec{F}$
 (b) Evaluate $\int_C x dy + y dx$ where C is the loop of the Folium of Descartes $x = \frac{3at}{1+t^3}$, $y = \frac{3at^2}{1+t^3}$ [8+7]

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1. (a) Find $L[3 \cos 3t \cos 4t]$
- (b) Find the inverse Laplace transform of $\log \left(1 + \frac{16}{s^2} \right)$ [7+8]
2. (a) Solve the differential equation $(D^4 - 2D^3 + 2D^2 - 2D + 1)y = \cos x$
- (b) Solve the differential equation $(D^3 - 3D - 2)y = x^2$ [7+8]
3. (a) Form the differential equation by eliminating arbitrary constants $\sin^{-1}x + \sin^{-1}y = C$.
- (b) Solve the differential equation $\frac{y}{x} \frac{dy}{dx} = \frac{P}{1 + x^2 + y^2x^2 + y^2}$.
- (c) Prove that the system of Parabolas $y^2 = 4a(x + a)$ is self orthogonal. [3+6+6]
4. (a) Apply Rolle's theorem for $\sin n \cos 2n$ in $0, \frac{\pi}{4}$ and find x such that $0 < x < \frac{\pi}{4}$
- (b) Expand $e^x \cdot \cos y$ near the point $1, \frac{\pi}{4}$ by Taylor's theorem. [7+8]
5. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$
- (b) Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2} \right)$ [8+7]
6. (a) Find the radius of curvature at the point on $x = a \log(\sec \theta + \tan \theta)$ and $y = a \sec \theta$
- (b) Trace the curve $x^3 + y^3 = 3axy$ [8+7]
7. (a) Prove that the surface area of the solid generated when the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is revolved about its major axis is $2ab \int_0^{\pi} \sqrt{1 - e^2 \sin^2 \theta} + \frac{\sin^{-1} e}{e}$ where e is the eccentricity of the ellipse.
- (b) Evaluate $\iiint_R (xy + yz + zx) dx dy dz$, where V is the region of space bounded by $x=0, x=1, y=0, y=2$ and $z=0, z=3$ [7+8]
8. Verify Stoke's theorem for $F = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$ over upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plane. [15]

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1. (a) Find $L \left[t e^{2t} \sin 3t \right]_i$
 (b) Find $L^{-1} \frac{1}{s^3(s^3+1)}$ [8+7]
2. (a) By considering the function $(x-2) \log x$ show that the equation $x \log x = 2-x$ is satisfied by at least one value of x lying between 1 and 2.
 (b) Find the minimum of $x^2 + y^2 + z^2$ subject $x + y + z = 3a$ [7+8]
3. (a) Find the volume of the solid obtained by revolving one arch of the cycloid $x = a \left(\theta + \sin \theta \right)$, $y = a \left(1 - \cos \theta \right)$ about its base.
 (b) Calculate $\int_R r^3 dr d\theta$ over the area included between the circles $r = 2 \sin \theta$ and $r = 4 \sin \theta$. [8+7]
4. (a) If \vec{F} and \vec{G} are two vectors, then $\text{div } \vec{F} \times \vec{G} = \vec{F} \cdot \text{curl } \vec{G} - \vec{G} \cdot \text{curl } \vec{F}$
 (b) Evaluate by Greens theorem $\int_C (x^2 - \cos y) dx + (y + \sin x) dy$ where C is the rectangle with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$ [8+7]
5. (a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2n!}{n!(n)}$
 (b) Test the convergence of the series $= \frac{2.5.8 \dots 3n-1}{1.5.9 \dots 4n-3}$
 (c) Find the interval of convergence for the following series $\sum_{n=1}^{\infty} \frac{(n^2-1)}{n^2+1} x^n$. [5+5+5]
6. (a) If CP and CD are a pair of conjugate diameters of an ellipse prove that the radius of curvature at P is $\frac{(CD)^3}{ab}$ a and b being the lengths of the semiarcs of the ellipse.
 (b) Trace the curve $y^2 = x^2 \frac{(3a-x)}{(a+x)}$ [8+7]
7. (a) Find the differential equation of all circles which pass through the origin and whose centers are on x- axis.
 (b) Solve the differential equation $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$
 (c) The rate at which bacteria multiply is proportional to the instantaneous N numbers present. If the original number doubles in 2hrs? When it will be trebled? [4+6+5]
8. (a) Solve the differential equation $(D^2 - 1)y = x \sin x + (1 + x^2)e^x$

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(b) Solve the differential equation $(D^2 + 4)y = \text{Tan}2x$

[8+7]

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